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## Power-Thermal Modeling of 3-phase Switching Amplifiers

Thermal modeling in a 3-phase inverter is difficult because there are 12 devices all creating significant amounts of heat. In comparison, a high-current linear amplifier only has 2 heat-dominating devices. With this added complexity, 3 -phase designs are even more prone to mistakes, leading to catastrophic failure from insufficient heat sinking. At the same time, modeling all 12 heat-producing devices can be overwhelming in (what should be) a simple motor-drive application. The following article explains how to simplify the model and optimize the heatsink for the application.

## TYPES OF POWER LOSSES

Power in any switching amplifier consists of several parts. The most obvious part is conduction losses: the power generated in each switch when a switch is on. We assume that the switch has some electrical resistance which, when subjected to a current, produces heat. We also assume that when each switch is "open," the current through it is zero, and there is negligible power dissipated in this state.

Figure 1: General Schematic for a MOSFET-based inverter circuit


Similar to conduction losses, we have diode losses. This refers to the free-wheeling diodes that conduct current in the opposite direction of the switches (see Figure 1). Often, the free-wheeling diodes are simply the parasitic body diodes of the MOSFET or Bipolar Transistor that serves as the switch. However, it is sometimes wise to parallel these body diodes with discrete diodes for lower losses, faster response times, or better heat spreading. Some Apex devices, such as SA310, have discrete free-wheeling diodes built-in to the amplifier itself.

Another significant power loss is switching loss. When a switch closes, it takes time for the current to "ramp up" to the final value. During this time, the voltage across the switch is theoretically ramping down to its final value. That means there is a period of time when both the current and voltage are significantly far from zero, leading to an enormous instantaneous power dissipation. These instantaneous "spikes" average out over time and sum up to the switching losses.

Figure 2: Switching Losses


Lastly, losses in the gate driver circuitry contribute to the " $\mathrm{V}_{\mathrm{CC}}$ losses". Typically, this power is insignificant compared to the total. However, it can be easily calculated as the product of $\mathrm{V}_{\mathrm{CC}}$ and the current going into the $\mathrm{V}_{\mathrm{CC}}$ pin.

## POWER EQUATIONS

The equations below apply to a system with the following conditions:
1.A sinusoidal commutation style is used.
2.The motor is not changing speed or load.
3. The motor is not stalling.
4. I RIPPLE $^{\ll I_{\text {PEAK }} \text { (see equations to calculate). }}$
$5 . Z_{\text {WYE }} \gg R_{\text {DS(ON) }}$ (see variable descriptions).

$$
\begin{aligned}
& I_{\text {RIPPLE }}=\frac{4 D C_{M A X}}{Z_{\text {WYEFFSW }}} \times\left(\frac{V_{S}}{2}-V_{E M F}\right) \\
& I_{\text {PEAK }}=\frac{V_{S} D C_{M A X}-V_{E M F}}{Z_{W Y E}} \\
& P_{C O N D, E A C H}=I_{P E A K}^{2} R_{D S(O N)}\left(\frac{16 D C_{M A X} \cos \left(\theta_{W Y E}\right)+3 \pi}{24 \pi}\right) \\
& P_{\text {DIODE }, \text { ACH }}=I_{\text {PEAK }} R_{\text {DIODE }} \frac{3 \pi I_{\text {PEAK }}+12 \frac{V_{\text {DIODE }}}{R_{\text {DIODE }}}-D C_{M A X} \cos \left(\theta_{W Y E}\right)\left(6 \pi \frac{V_{\text {DIODE }}}{R_{\text {DIODE }}}+16 I_{\text {PEAK }}\right)}{24 \pi} \\
& P_{\text {SWITCHING,TOTAL }}=\frac{3}{\pi} V_{S} I_{\text {PEAK }} f_{S W}\left(t_{\text {RISE }}+t_{\text {FALL }}\right) \\
& P_{C C}=V_{C C} I_{C C} \\
& P_{\text {INVERTER,TOTAL }}=6\left(P_{\text {COND,EACH }}+P_{\text {DIODE,EACH }}\right)+P_{\text {SWITCHING,TOTAL }}+P_{C C} \\
& P_{\text {LOAD }}=\frac{3}{2} I_{\text {PEAK }} V_{S} D C_{M A X} \cos \left(\theta_{\text {WYE }}\right) \\
& I_{S, A V G}=\frac{P_{L O A D}+6\left(P_{\text {COND,EACH }}+P_{\text {DIODE,EACH }}\right)+P_{\text {SWITCHING }, T O T A L}}{V_{S}}
\end{aligned}
$$

## User-Selected Variables:

- $\mathrm{V}_{\mathrm{S}}=$ Supply Voltage for high-current outputs.
- $\quad \mathrm{V}_{\mathrm{CC}}=$ Supply Voltage for gate drive circuitry. The amplifier typically has a suggested value for this.
- $\mathrm{f}_{\mathrm{SW}}=$ Switching Frequency.
- $\quad D C_{\text {MAX }}=$ Maximum Duty Cycle deviation from a $50 \%$ baseline. Value should be between 0 and 0.5 (See figure 3). A higher $\mathrm{DC}_{\text {MAX }}$ is used to apply more current into the load

Figure 3: $\mathrm{DC}_{\text {MAX }}$ Definition, displaying $\mathrm{DC}_{\text {MAX }}=0.25$


Variables specified by amplifier datasheet:

- $\mathrm{I}_{\mathrm{CC}}=$ Supply Current for gate drive circuitry. This is usually a function of $\mathrm{f}_{\mathrm{SW}}$.
- $R_{D S(O N)}=$ Switch resistance when closed. This is a function of junction temperature.
- $R_{\text {DIODE }}, V_{\text {DIODE }}$ = fictitious values used to approximate the diode behavior when forward biased. See figure 4 to derive these values. For Apex SA310 modules, the diodes would have values $\mathrm{R}_{\text {DIODE }}=22 \mathrm{~m} \Omega$ and $\mathrm{V}_{\mathrm{DI}}$ ODE $=0.9 \mathrm{~V}$.
- $\mathrm{t}_{\text {RISE }}, \mathrm{t}_{\text {FALL }}=$ The rise and fall time [s] for a complete transition. These are assumed constant.

Figure 4: Diode Model


Variables specified by the motor:

- $\mathrm{V}_{\mathrm{EMF}}=$ Peak amplitude of the Back-EMF voltage produced by one leg of a wye-equivalent motor. $\mathrm{V}_{\mathrm{EMF}}$ is a function of drive frequency.

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- $\mathrm{Z}_{\mathrm{WYE}}, \theta_{\mathrm{WYE}}=$ Impedance $[\Omega]$ and Phase Angle of a wye-equivalent leg of the load, evaluated at the desired drive frequency. See figure 5 for converting between wye- and delta- connected loads.
- $\quad Z_{W Y E, F S W}=$ Impedance [ $\Omega$ ] of a wye-equivalent leg of the load, evaluated at the switching frequency. This impedance should be much higher than that of $Z_{\text {WYE }}$. See figure 5 for converting between wye- and deltaconnected loads.
- $R_{\text {WYE }}=D C$ resistance of a wye-equivalent leg of the load. See figure 5 for converting between wye- and delta- connected loads.

Figure 5: Delta-Wye Conversion Chart


## Calculated Values:

- $\mathrm{I}_{\text {RIPPLE }}=$ Ripple current, in Amps Peak-to-Peak.
- $I_{\text {PEAK }}=$ Peak Amplitude of current into one phase of the load, ignoring ripple current.
- $P_{\text {COND,EACH }}=$ Conduction Power in each of the 6 switches, averaged over time.
- $P_{\text {DIODE,EACH }}=$ Diode Power in each of the 6 free-wheeling diodes, averaged over time.
- $P_{\text {SWITCHING,Total }}=$ Total Switching Power in the 3-phase inverter, averaged over time.
- $\mathrm{P}_{\mathrm{CC}}=$ Total $\mathrm{V}_{\mathrm{CC}}$ losses.
- $P_{\text {inverter,Total }}=$ Total losses in the 3-phase inverter, averaged over time.
- $P_{\text {LOAD }}=$ Total power dissipated in the motor, averaged over time.
- $\mathrm{I}_{\mathrm{S}, \mathrm{AVG}}=$ Average supply current from the $\mathrm{V}_{\mathrm{S}}$ supply at steady-state. The $\mathrm{V}_{\mathrm{S}}$ supply must contribute this much current during steady-state. Rapid starts, stops, and reversals will demand more current from $\mathrm{V}_{\mathrm{S}}$.

See Appendix A for a design example.
Conditions that are not covered here, such as trapezoidal/six-step commutation, reversals, or stalls, should be solved in an automated simulation tool, such as Apex Power Design or a SPICE simulation tool.

SPICE simulation is certainly a great tool for calculating power in inverters. However, the accuracy of switching losses is entirely dependent on the time stepping function. Different SPICE engines, which each have their own time stepping functions, can produce vastly different answers for the same circuit. In addition, it may be difficult and/or time consuming to model the electro-thermal mutual dependence in a SPICE
simulator. The above equations, when applicable, give a much faster and more consistent result. Apex Power Design calculates switching losses independently via the equation above.

## THERMAL MODEL

A simple thermal model can be made using electrical "elements". In a nutshell, voltage in the model represents Temperature in real life. Likewise, current represents power dissipation and/or heat transfer, and ohmic resistance represents thermal resistance. More information on this style of thermal modeling can be found in AN01.

The equations above provide all the power dissipation values for use in the thermal model of figure 6. Thermal resistances are also known - these are component properties:

- Junction-to-Case Thermal Resistance ( $R_{\theta J C}$ ) - given in amplifier datasheet.
- Case-to-Sink Thermal Resistance ( $R_{\theta C S}$ ) - property of Thermal Interface Material (TIM).
- Sink-to-Air Thermal Resistance ( $R_{\theta S A}$ ) - given in heatsink drawing. Decreases with air (or water) flow rate.

Figure 6: Thermal Model Construction


For three-phase inverters, junction-to-case thermal resistance can be confusing because there are essentially 12 junctions ( 6 switches and 6 diodes). Apex combines the thermal resistance of the 6 switches into one thermal resistance number published in the datasheet. Additionally, on relevant devices, Apex combines the thermal resistance of the 6 diodes into one separate number in the datasheet. These numbers would be plugged into $\theta_{J C, S W I T C H}$ and $\theta_{J C, D I O D E}$ respectively in the model of figure 6.

## MUTUAL DEPENDENCE

Recall that most switch technologies (MOSFETs, IGBTs) have a resistance that increases with the junction temperature. That makes it difficult to estimate the true $R_{D S(O N)}$ in the equations above, as the junction temperature is not yet known. To get the true $R_{D S(O N)}$, we must first guess the $R_{D S(O N)}$ and solve for the Total Inverter Power Dissipation. Once we have the power dissipation, enter it into the thermal model for the module (see Thermal Model section). The thermal model will, among other things, reveal the junction tempera-
ture. We can then use that junction temperature to make a better guess at the $\mathrm{R}_{\mathrm{DS}(\mathrm{ON})}$ and repeat the process.

A possible iteration table may look like that of figure 7. The solution will slowly converge on a final junction temperature, establishing the correct $\mathrm{R}_{\mathrm{DS}(\mathrm{ON})}$ and Power Dissipation. Most models converge (reach a relative error of $0.1 \%$ or less) in under 10 iterations.

Figure 7: Example Iteration Table


In some cases, the solution does not converge; instead, the junction temperature increases more and more with each iteration. This is thermal runaway, a good indication that the conditions are too stressful for the inverter module under consideration. In some cases, additional heatsinking (lower heatsink thermal resistance) can pull the thermal runaway back into stable convergence. Most cases of thermal runaway, however, require a more heavy-duty amplifier.

Silicon carbide devices, such as SA310 and SA110, are much more immune to thermal runaway. This is because SiC offers a lower temperature coefficient; $\mathrm{R}_{\mathrm{DS}(\mathrm{ON})}$ is less affected by temperature in SiC than it is for silicon switches. This also makes Apex's silicon carbide series well-suited for applications with widely-varying ambient temperatures.

## APPENDIX A: DESIGN EXAMPLE

A Wye connected PMSM, with each phase representing $2 \Omega$ and 4.7 mH , is driven by an SA310 inverter module at a 400 V supply voltage. At normal conditions, the motor will spin at 600 RPM and requires half of the full supply voltage. The motor has 5 poles ( 5 "electrical" rotations gives 1 mechanical rotation), and at the desired 600 RPM, the Back-EMF (sinusoidal) was measured at 45Vpeak per phase. Switching frequency is selected at 31.25 kHz and $\mathrm{V}_{\mathrm{CC}}$ is at the inverter's nominal value (18V).

First, we need to calculate the motor's wye-impedance at both the cycle frequency ( 600 RPM mechanical) and the switching frequency ( 31.25 kHz )

$$
\begin{gathered}
f_{C Y C L E}=\frac{R P M \cdot n_{P O L E S}}{60 \frac{s}{m i n}}=\frac{600 R P M(5)}{60 \frac{s}{\min }}=50 \mathrm{~Hz} \\
Z_{W Y E}=\sqrt{R_{W Y E}^{2}+\left(2 \pi f_{C Y C L E} L_{W Y E}\right)^{2}}=\sqrt{(2 \Omega)^{2}+(2 \pi(50 \mathrm{~Hz})(4.7 \mathrm{mH}))^{2}}=2.486 \Omega \\
\theta_{W Y E}=\tan ^{-1}\left(\frac{2 \pi f_{C Y C L E} L_{W Y E}}{R_{W Y E}}\right)=\tan ^{-1}\left(\frac{2 \pi(50 \mathrm{~Hz})(4.7 \mathrm{mH}))}{2 \Omega}\right)=36.44^{\circ} \\
Z_{W Y E, F S W}=\sqrt{R_{W Y E}^{2}+\left(2 \pi f_{S_{W I T C H I N G}{ }^{\circ} W Y E}\right)^{2}}=\sqrt{(2 \Omega)^{2}+(2 \pi(31.25 \mathrm{kHz})(4.7 \mathrm{mH}))^{2}}=922.8 \Omega
\end{gathered}
$$

So far, we have passed one requirement for the power equation validity test: that $Z_{\text {WYE }}$ is at least an order of magnitude higher than $R_{D S(O N)}=43 \mathrm{~m} \Omega$. Now let's check the ripple current. For that, we need to understand that $\mathrm{DC}_{\text {MAX }}$ (in the sinusoidal case only) is the peak duty cycle from a $50 \%$ baseline. Since we only need half of the full supply voltage, the duty cycle would vary from $25 \%$ to $75 \%$ on each phase. Therefore, $\mathrm{DC}_{\text {MAX }}$ is $25 \%$

$$
\begin{gathered}
I_{R I P P L E}=\frac{4 D C_{M A X}}{Z_{W Y E, F S W}}\left(\frac{V_{S}}{2}-V_{E M F}\right)=\frac{4(0.25)}{(922.8 \Omega)}\left(\frac{(400 \mathrm{~V})}{2}-(45 \mathrm{~V})\right)=168 \mathrm{~mA} \\
I_{P E A K}=\frac{\left(V_{S} D C_{M a x}-V_{E M F}\right)}{Z_{W Y E}}=\frac{((400 \mathrm{~V})(0.25)-(45))}{(2.486 \Omega)}=22.12 \mathrm{~A}
\end{gathered}
$$

This passes the other validity test $\left(I_{\text {RIPPLE }} \ll I_{\text {PEAK }}\right)$, so we know the power equations will yield results very close to reality.
$R_{D S(O N)}$ changes with junction temperature, which we don't know yet. However, we can give an educated guess and iterate these equations later until we converge on the correct junction temperature. Let's start with a guess of $R_{D S(O N)}=48 \mathrm{~m} \Omega$.

$$
\begin{aligned}
& P_{C O N D, E A C H}=I_{P E A K}^{2} R_{D S(O N)}\left(\frac{16 D C_{M A X} \cos \left(\left(\theta_{W Y E}\right)+3 \pi\right)}{24 \pi}\right) \\
& =(22.12 \mathrm{~A})^{2}(48 m \Omega)\left(\frac{16(0.25) \cos \left(36.44^{\circ}\right)+3 \pi}{24 \pi}\right)=3.938 \mathrm{~W} \\
& P_{\text {DIODE,EACH }}=I_{P E A K} R_{D I O D E} \frac{3 \pi I_{P E A K}+12\left(\frac{V_{D I O D E}}{R_{\text {DIODE }}}\right)-D C_{M A X} \cos \left(\theta_{W Y E}\right)\left(6 \pi \frac{V_{D I O D E}}{R_{D I O D E}}+16 I_{P E A K}\right)}{24 \pi} \\
& =(22.12 A)(22 m \Omega) \frac{3 \pi(22.12 A)+12\left(\frac{0.9 V}{22 m \Omega}\right)-(0.25) \cos \left(36.44^{\circ}\right)\left((6 \pi) \frac{0.9 V}{22 m \Omega}+16(22.12 A)\right)}{24 \pi}=3.054 \mathrm{~W} \\
& \left.P_{\text {SWITCHING,TOTAL }}=\frac{3}{\pi} V_{S} I_{\text {PEAK }} f_{S W}\left(t_{\text {Rise }}+t_{\text {Fall }}\right)=\frac{3}{\pi} 400 \mathrm{~V}\right)(22.12 \mathrm{~A})(31.25 \mathrm{kHz})(45 \mathrm{~ns}+30 \mathrm{~ns})=19.8 \mathrm{~W} \\
& P_{c c}=V_{c c} I_{c c}=(18 V)(13 m A)=0.234 W \\
& P_{\text {INVERTER,TOTAL }}=6\left(P_{\text {COND,EACH }}+P_{\text {DIODE,EACH }}\right)+P_{\text {SWITCHING,TOTAL }}+P_{C C} \\
& =6(3.938 W+3.054 W)+19.8 W+0.234 W=62 W \\
& P_{L O A D}=\frac{3}{2} I_{P E A K} V_{S} D C_{M A X} \cos \left(\theta_{W Y E}\right)=\frac{3}{2}(22.12 A)(400 \mathrm{~V})(0.25) \cos (36.44)=2.67 \mathrm{~kW} \\
& =3.58 \mathrm{hp} \\
& I_{S, A V G}=\frac{P_{L O A D}+6\left(P_{C O N D, E A C H}+P_{\text {DIODE,EACH }}\right)+P_{\text {SWITCHING }}}{V_{S}} \\
& =\frac{2.669 \mathrm{~kW}+6(3.938 \mathrm{~W}+3.054 \mathrm{~W})+19.8 \mathrm{~W}}{(400 \mathrm{~V})}=6.827 \mathrm{~A}
\end{aligned}
$$

Notice that the SA310 dissipates 62W in this scenario, which is certainly possible from the SA310 with some moderate heatsinking. The next step is to estimate the junction temperature with this power and heatsinking, use that to estimate the true $\mathrm{R}_{\mathrm{DS}(\mathrm{ON})}$, and re-calculate the conduction losses. Iterate this process until the solution converges satisfactorily.

Also notice that the average supply current (6.8A) is much lower than the peak current (22.1A), or even the RMS value of the peak current (15.6A). This is no arithmetic error; this is due to the inductive nature of the load. The high inductance in this motor forces a significant fraction of the current into the free-wheeling diodes, where it then charges the $\mathrm{V}_{\mathrm{S}}$ supply bypass capacitors. This assumes the bypass capacitance on $\mathrm{V}_{\mathrm{S}}$ is at least large enough to store all the energy that can be stored in the load inductance. The more inductive the load is, the lower the $\mathrm{V}_{\mathrm{S}}$ supply current will be at steady-state.

## APPENDIX B: DERIVATION

Calculating Power Dissipation in an Apex switching amplifier consists of 3 steps:

1. Define the currents for each output
2. Determine when each switch and diode is conducting
3. Use $\mathrm{P}=\mathrm{IV}$ to find the average power dissipation in each device.

## Step 1: Defining Currents:

For sinusoidal commutation, defining the current relies on the principal of superposition - that the current will be the sum of a large, perfect sinewave, and a "ripple current", as shown in figure 8.

Figure 8: Superposition of Ideal and Ripple Currents


For simplicity, we assume the ripple current is small enough that it does not significantly impact the conduction losses. This is where condition \#4 comes from - ripple current must be small compared to the overall phase current amplitude.

The other 2 phases may be assumed identical in amplitude, with phase shifts of $120^{\circ}$ and $240^{\circ}$.
Although the 3 output voltages are pulse-width-modulated signals and not sinewaves, we can equate the output voltages to equivalent sinewaves that would produce the ideal current waveforms. This is easy to do the voltage can simply be represented as the duty cycle of a given output times $\mathrm{V}_{\mathrm{S}}$. Note that this ignores the voltage drop across the switches - condition \#5 implies that this voltage drop is insignificant.

$$
V_{\text {OUT,EQUIVALENT }}=V_{S} D C
$$

Duty cycle can further be expressed as a function of our selected $D C_{M A X}$. Recall that $D C_{M A X}$ is defined as the amplitude from a $50 \%$ baseline, meaning if $D C_{M A X}=50 \%$, the duty cycle will vary from $0 \%$ to $100 \%$.

$$
\begin{gathered}
D C=\left(0.5+D C_{M A X} \sin \left(2 \pi f_{C Y C L E} t\right)\right) \\
V_{\text {OUT,EQUIVALENT }}=V_{S}\left(0.5+D C_{M A X} \sin \left(2 \pi f_{C Y C L E} t\right)\right)
\end{gathered}
$$

From here, we continue with the Wye-equivalent form of the load:

Figure 9: Wye Load


In our equivalent-voltage model, the Wye center voltage can be proven to be $1 / 2 \mathrm{VS}$ by applying two principles:

1. The sum of $I_{U}, I_{V}$, and $I_{w}$ is 0 .
2.The sum of 3 sines of equal magnitude and equal phase distribution ( $0^{\circ}, 120^{\circ}$, and $240^{\circ}$ ) is 0 .

$$
\begin{aligned}
& V_{\text {OUT,U }}=\left(R_{W Y E} I_{U}+L_{W Y E} \frac{d I_{U}}{d t}+V_{E M F, U}+V_{\text {CENTER }}\right) \\
& V_{\text {OUT,V }}=\left(R_{W Y E} I_{V}+L_{W Y E} \frac{d I_{U}}{d t}+V_{E M F, V}+V_{\text {CENTER }}\right) \\
& V_{\text {OUT,W }}=\left(R_{W Y E} I_{W}+L_{W Y E} \frac{d I_{U}}{d t}+V_{E M F, W}+V_{\text {CENTER }}\right)
\end{aligned}
$$

$+$

$$
\begin{gathered}
\sum V_{O U T, X}=R_{W Y E} \sum I_{X}+L_{W Y E} \sum \frac{d I_{x}}{d t}+\sum V_{\text {EMF, } X}+3 V_{\text {CENTER }} \\
V_{\text {OUT,U,EQUIVALENT }}+V_{\text {OUT,V,EQUIVALENT }}+V_{\text {OUT,W,EQUIVALENT }}=3 V_{\text {CENTER,EQUIVALENT }} \\
\left.3 V_{S}(0.5)+3 V_{S} D C_{M A X}\left(\sin (\omega t)+\sin \omega t+120^{\circ}\right)+\sin \left(\omega t+240^{\circ}\right)\right)=3 V_{\text {CENTER,EQUIVALENT }} \\
1.5 V_{S}=3 V_{\text {CENTER,EQUIVALENT }} \\
V_{\text {CENTER,EQUIVALENT }}=\frac{1}{2} V_{S}
\end{gathered}
$$

This gives us an equivalent phase voltage:

$$
V_{\text {PHASE,EQUIVALENT }}=V_{\text {OUT,EQUIVALENT }}-V_{\text {CENTER,EQUIVALENT }}=V_{S} D C_{M A X} \sin \left(2 \pi f_{\text {CYCLE }} t\right)
$$

Finally, we can solve for the phase currents using impedance and phase-angle notation:

$$
\begin{gathered}
I_{P H A S E, I D E A L}=\frac{V_{P H A S E, E Q U I V A L E N T}-V_{E M F, X}}{Z_{W Y E} \angle \theta_{W Y E}} \\
I_{P H A S E, I D E A L}=\frac{V_{S} D C_{M A X} \sin \left(2 \pi f_{C Y C L E} t\right)-V_{E M F} \sin \left(2 \pi f_{C Y C L E} t\right)}{Z_{W Y E} \angle \theta_{W Y E}} \\
I_{P H A S E, I D E A L}=\frac{V_{S} D C_{M A X}-V_{E M F}}{Z_{W Y E}} \sin \left(2 \pi f_{C Y C L E} t-\theta_{W Y E}\right)
\end{gathered}
$$

Moving forward, we want to know the peak current of each phase. This can be found by evaluating the above equation where sine is at its maximum value.

$$
I_{P E A K}=\frac{V_{S} D C_{M A X}-V_{E M F}}{Z_{W Y E}}
$$

## Step 2: Determine when each switch and diode is conducting:

Recall that the real output voltages are pulse-width-modulated signals shifting between ground and $\mathrm{V}_{\mathrm{S}}$. Referring back to this definition, the following table defines the currents for devices of an individual phase:

|  | $\ldots . \mathrm{V}_{\text {OUT }}$ is... | ...And $\mathrm{I}_{\text {OUT }}$ is... |
| :--- | :--- | :--- |
| High-Side MOSFET is conducting when... | $\mathrm{V}_{\text {S }}$ | positive |
| Low-Side MOSFET is conducting when... | GND | negative |
| High-Side Diode is conducting when... | $\mathrm{V}_{\text {S }}$ | negative |
| Low-Side Diode is conducting when... | GND | positive |

By masking the equation for current from step 1 with the above conditions, a typical MOSFET current would look like the that in figure 10:

Figure 10: MOSFET Current


Note that the maximum duty cycle does not always coincide with the maximum current. Instead, they are separated by a phase lag $\theta_{\text {WYE }}$, as expected between the current and voltage waveforms.

The MOSFET current waveform in figure 10 does not easily form itself into a simple equation; this is why we will want to express the MOSFET current as a current "envelope" and a duty cycle "width mask", as in figure 11. These expressions will come in very handy in step 3.

Figure 11: Width Mask and Envelope representations of MOSFET Current


For this representation, we have simplified the sine function to be a function of $\theta$ rather than t , where:

$$
\theta=2 \pi f_{C Y C L E} t-\theta_{W Y E}
$$

During normal operation, the six switches will have current waveforms that are mirrored and phaseshifted from each other. None of these transformations will affect the overall power in each switch, so it is sufficient to assume that the power calculated in one switch represents the power dissipated in any switch. Therefore, this derivation does not need to continue finding expressions for the currents in the remaining 5 switches. However, abnormal conditions such as motor stalls must consider that power is not distributed evenly between all 6 switches.

Similarly, the current waveform of one diode can represent that of all 6 diodes. Using the above table, the current in a typical diode will look like that in figure 12:

Figure 12: Diode Current


Envelope and width-mask notation for the diode is very similar; the duty cycle is merely inverted from that of the MOSFET's expression:

Envelope:

$$
I_{E N V, D I O D E}=\left\{\begin{array}{c}
I_{P E A K} \sin (\theta), 0<\theta \leq \pi \\
0, \pi<\theta \leq 2 \pi
\end{array}\right.
$$

With Mask:

$$
D C_{D I O D E}=0.5-D C_{M A X} \sin \left(\theta+\theta_{W Y E}\right)
$$

## Step 3: Find Average Power Dissipation:

Here comes the heavy math part.
The switching nature of the currents in the amplifier makes this calculation amenable to a Reimann sum we can treat each current "pulse" as a rectangle, calculate the energy produced within each rectangle, sum up those energies, and divide by the period to get an average power. To make this process generic to any "pulse" frequency, we can calculate the Reimann sum as a definite integral. For the MOSFET, that integral will look like:

$$
\begin{gathered}
P_{C O N D, E A C H}=\frac{1}{2 \pi} \int_{0}^{\pi} P_{E N V, M O S F E T} d \theta^{\prime} \\
P_{C O N D, E A C H}=\frac{1}{2 \pi} \int_{0}^{\pi} I_{E N V, M O S F E T}^{2} R_{D S(O N)} d \theta^{\prime}
\end{gathered}
$$

By limiting the period of integration from 0 to $\pi$ (instead of $2 \pi$ ), the piece-wise nature of the envelope function is considered. The variable of integration ( $\mathrm{d} \theta^{\prime}$ ) must be modified by the width mask function. For the MOSFET, this would look like:

$$
d \theta^{\prime}=D C_{M O S F E T} d \theta=\left(0.5+D C_{M A X} \sin \left(\theta+\theta_{W Y E}\right)\right) d \theta
$$

Substituting everything in:

$$
P_{C O N D, E A C H}=\frac{1}{2 \pi} \int_{0}^{\pi}\left(I_{P E A K} \sin (\theta)\right)^{2} R_{D S(O N)}\left(0.5+D C_{M A X} \sin \left(\theta+\theta_{W Y E}\right)\right) d \theta
$$

This Integral has a solution. It is:

$$
P_{C O N D, E A C H}=I_{P E A K}^{2} R_{D S(O N)}\left(\frac{16 D C_{M A X} \cos \left(\theta_{W Y E}\right)+3 \pi}{24 \pi}\right)
$$

The power for the diode is a bit different, since we need to know the diode voltage. Luckily, we are only concerned with the diode behavior when the current is positive through the diode, so we can approximate the voltage with a straight line as shown in figure 4. This gives diode forward voltage as a function of current:

$$
V_{F}=\left\{\begin{array}{rr}
V_{\text {DIODE }}+R_{\text {DIODE }} \times I_{E N V, D I O D E} & I_{E N V, D I O D E \geq 0} \\
\text { Irrelevant } & I_{E N V, D I O D E<0}
\end{array}\right.
$$

The power-envelope expression for the diode will be:

$$
P_{E N V, D I O D E}=I_{E N V, D I O D E} \times V_{F}=I_{E N V, D I O D E} V_{D I O D E}+I_{E N V, D I O D E}^{2} R_{D I O D E}
$$

Once again, We construct an integral, this time with the diode power envelope and width mask functions:

$$
P_{D I O D E, E A C H}=\frac{1}{2 \pi} \int_{0}^{\pi} P_{E N V, D I O D E} d \theta^{\prime}
$$

$P_{\text {DIODE,EACH }}=\frac{1}{2 \pi} \int_{0}^{\pi}\left(I_{E N V, D I O D E} V_{D I O D E}+I_{E N V, D I O D E}^{2} R_{D I O D E}\right)\left(D C_{D I O D E} d \theta\right)$
$P_{\text {DIODE }, E A C H}=\frac{1}{2 \pi} \int_{0}^{\pi}\left(I_{\text {PEAK }} V_{D I O D E} \sin (\theta)+I_{P E A K}^{2} R_{D I O D E} \sin ^{2}(\theta)\right)\left(0.5-D C_{M A X} \sin \left(\theta+\theta_{W Y E}\right)\right) d \theta$

The Solution to this integral is:

$$
P_{D I O D E, E A C H}=I_{P E A K} R_{D I O D E} \frac{3 \pi I_{P E A K}+12 \frac{V_{D I O D E}}{R_{D I O D E}}-D C_{M A X} \cos \left(\theta_{W Y E}\right)\left(6 \pi \frac{V_{D I O D E}}{R_{D I O D E}}+16 I_{P E A K}\right)}{24 \pi}
$$

So far we have neglected switching losses. We have assumed each switch is always either at its maximum or minimum impedance. However, as figure 2 shows, this is not always true.

We cannot assume that each transition will be a completely linear path from high-to-low or low-to-high. Switching behavior, even when optimized, can have bends and non-linearities that make a large impact on the real vs. calculated switching losses. Additionally, the current and voltage transitions may not be perfectly synchronized. The industry-accepted "worst case" switching power estimation is defined as:

$$
P_{S W}=\frac{1}{2} I_{S W} V_{S}
$$

Where $I_{\text {SW }}$ is defined as the magnitude of current that is switched on or off. In our case, $I_{\text {SW }}$ will follow the current envelope that we have defined for the MOSFET, $I_{\text {ENV,MOSFET }}$

$$
I_{S W}=I_{E N V, M O S F E T}=I_{P E A K} \sin (\theta)
$$

Remember that this equation is only valid for the duration of switching times $\mathrm{t}_{\text {FALL }}$ and $\mathrm{t}_{\text {RISE }}$. We must once again make a Reimann sum to average out the switching losses given our specific current waveform and switching frequency. Just like the conduction losses, we can do this in the form of a definite integral.

Figure 13: Switching Current for 1 phase, Repeating every half period


For this integral, we will only focus on half a period. Switching current $\mathrm{I}_{\mathrm{SW}}$ should repeat nearly identically every half period. See figure 13.

$$
P_{\text {SWITCHING,1PHASE }}=\frac{1}{\pi} \int_{0}^{\pi} P_{S W} d \theta^{\prime}
$$

Where $d \theta$ ' only integrates over the rise time ( $\mathrm{t}_{\text {RISE }}$ ) and fall time ( $\mathrm{t}_{\mathrm{FALL}}$ ) once each per switching period ( $T_{s w}$ ).

$$
\begin{gathered}
d \theta^{\prime}=\frac{t_{R I S E}+t_{F A L L}}{T_{S W}} d \theta=\left(t_{R I S E}+t_{F A L L}\right) f_{S W} d \theta \\
P_{\text {SWITCHING,TOTAL }}=\frac{1}{\pi} \int_{0}^{\pi}\left(\frac{1}{2} V_{S} I_{P E A K} \sin (\theta)\right)\left(t_{R I S E}+t_{F A L L}\right) f_{S W} d \theta \\
P_{\text {SWITCHING,1PHASE }}=\frac{1}{\pi} V_{S} I_{\text {PEAK }}\left(t_{R I S E}+t_{F A L L}\right) f_{S W}
\end{gathered}
$$

With 3 Phase Switching, the total losses will be triple:

$$
P_{\text {SWITCHING,TOTAL }}=\frac{3}{\pi} V_{S} I_{\text {PEAK }}\left(t_{\text {RISE }}+t_{F A L L}\right) f_{S W}
$$

Lastly, load power is often desired as a sanity check or to size the $\mathrm{V}_{\mathrm{S}}$ power supply to the correct wattage. Referring to figure 9 , the power dissipated in one phase of the load should be:

$$
P_{\text {LOAD,1PHASE }}=V_{\text {PHASE,EQUIVALENT }} I_{\text {PHASE }}
$$

$P_{\text {LOAD,1PHASE }}=\left(V_{S} D C_{M A X} \sin \left(2 \pi f_{C Y C L E} t\right)\right)\left(I_{P E A K} \sin \left(2 \pi f_{C Y C L E} t-\theta_{W Y E}\right)\right)$

Using the $\theta$-replacement technique to simplify:

$$
\begin{gathered}
P_{\text {LOAD,1PHASE }}=\left(V_{S} D C_{M A X} \sin \left(\theta+\theta_{W Y E}\right)\right)\left(I_{P E A K} \sin (\theta)\right) \\
P_{L O A D, 1 P H A S E}=V_{S} D C_{M A X} I_{P E A K} \sin \left(\theta+\theta_{W Y E}\right) \sin (\theta)
\end{gathered}
$$

The average power can be determined by integrating over one period:

$$
\begin{gathered}
P_{L O A D, 1 P H A S E, A V G}=\frac{1}{2 \pi} \int_{0}^{2 \pi} V_{S} D C_{M A X} I_{P E A K} \sin \left(\theta+\theta_{W Y E}\right) \sin (\theta) d \theta \\
P_{L O A D, 1 P H A S E, A V G}=\frac{1}{2} V_{S} D C_{M A X} I_{P E A K} \cos \left(\theta_{W Y E}\right)
\end{gathered}
$$

For 3 Phases, the total load power becomes:

$$
P_{L O A D}=\frac{3}{2} V_{S} D C_{M A X} I_{P E A K} \cos \left(\theta_{W Y E}\right)
$$

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